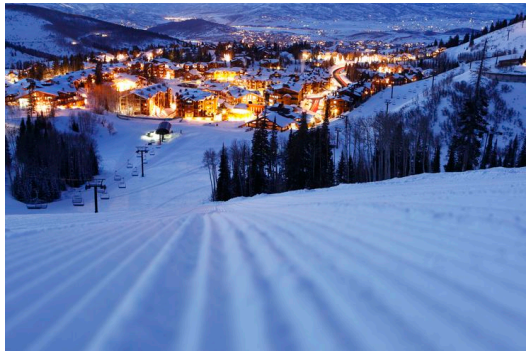
The Grass is Really Green on the Other Side

Empirical vs. Rigorous in Floating-Point Precision Analysis, Reproducibility, Resilience

Ganesh Gopalakrishnan

School of Computing, University of Utah, **Salt Lake City**, UT 84112



www. cs. utah. edu / fv

pruners.github.io

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collaborations with students, Utah colleagues
and LLNL (Dong Ahn), PNNL (Sriram Krishnamoorthy)

On Floating-Point Errors

Numerical errors are rare, rare enough
not to care about them all the time,
but yet not rare enough to ignore them.
— William M. Kahan

Empirical vs. Rigorous Methods

- Empirical (testing-based) methods are hugely important
  - "Beware of bugs in the above code; I have only proved it correct, not tried it."
    -- Knuth
- But do not give us insights into code behavior across all inputs
  - "Program **testing** can be used to show the **presence** of **bugs**, but never to show their absence!"
    -- Dijkstra
- Rigorous methods can help develop Better Empirical Methods
  - "We should continually be striving to transform every art into a science: in the process, we advance the art."
    -- Knuth

Empirical: "based on, concerned with, or verifiable by observation or experience rather than theory or pure logic"

Some Current Challenges in Floating-Point

- Floating-point code seldom carries rounding bounds
  - a rigorous guarantee of rounding error across intervals of inputs
  - Inferred specifications (rounding bounds extracted from code) can be useful
- Precision allocation is often done without rigorous guarantees
  - the resulting code may prove to be brittle
- Non-reproducibility (due to numerics) is a net productivity loss
  - code outlives hardware/compilers; answers may change after porting
- Soft errors can skew the numerics
  - long-running codes may harbor silent data corruptions
- Compiler bugs can also result in aberrant numerical results
  - compilers that reschedule operations are complex and have exhibited bugs

Unified Handling of Challenges

- We have developed rigorous approaches for roundoff error analysis

- These methods have proven valuable/promising to address "unrelated" challenges in detecting
  - soft errors, (published)
  - profiling precision, (in progress)
  - guarding against compiler bugs, (published)
  - and hopefully also reproducibility (TBD)

This Talk

- Scalable Rigorous Precision Estimation methods
  - Compute roundoff errors - tool SATIRE
    - Scalable Abstraction-guided Technique for Inferring Rounding Errors
  - Analytically bound roundoff errors - tool FPDetect
- Rigorous ways to catch soft errors ("bit flips")
  - FPDetect results in soft-error detectors that come with guarantees
  - Empirical ways to guard against polyhedral compiler bugs
    - FPDetect detectors can also help catch compiler bugs
- Empirical methods for reproducibility
  - Our FLiT tool can help isolate submodules thwarting reproducibility
  - Rigorous methods can help FLiT advance
- Conclusions

Coauthor and Funding credits

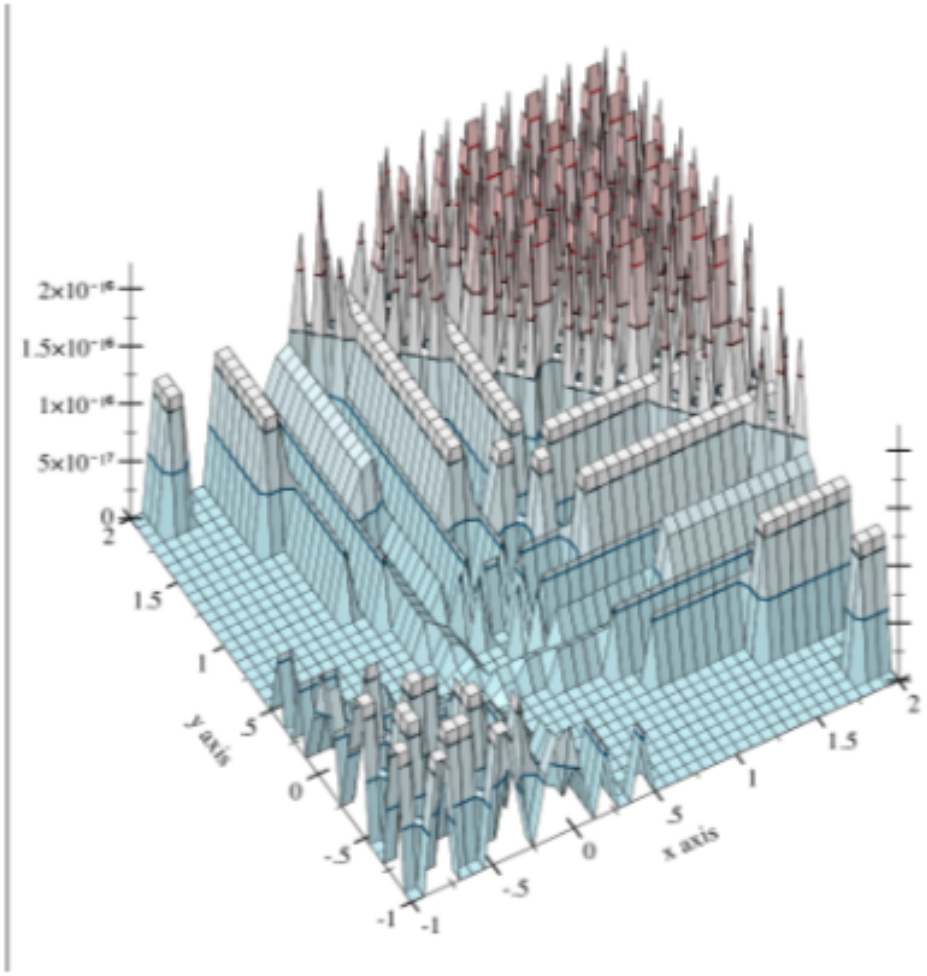
- SATIRE: Arnab Das (PhD stud), Ian Briggs (PhD stud), Sriram Krishnamoorthy (PNNL), Pavel Panchekha (U of U)
- FLiT: Michael Bentley (PhD stud), Ian Briggs (U), Dong H Ahn, Ignacio Laguna, Gregory L. Lee, Holger E. Jones (LLNL)
- FPDetect: Arnab Das, Ian Briggs, Sriram Krishnamoorthy, Ramakrishna Tipireddy (PNNL)
- Funding: NSF and DOE Contract (PNNL, LNNL)
  - NSF CCF – Awards: 1704715 , 1817073 , 1918497

Publications

- SATIRE
  - https://arxiv.org/abs/2004.11960
- FPDetect
  - making its way through TACO (ask us for a copy)
- FLiT
  - In HPDC'19 : https://dl.acm.org/doi/10.1145/3307681.3325960
  - To appear in CACM (ETA this year?)
  - https://github.com/PRUNERS

Let’s begin with roundoff error estimation

Nobody likes to use the actual Rounding error



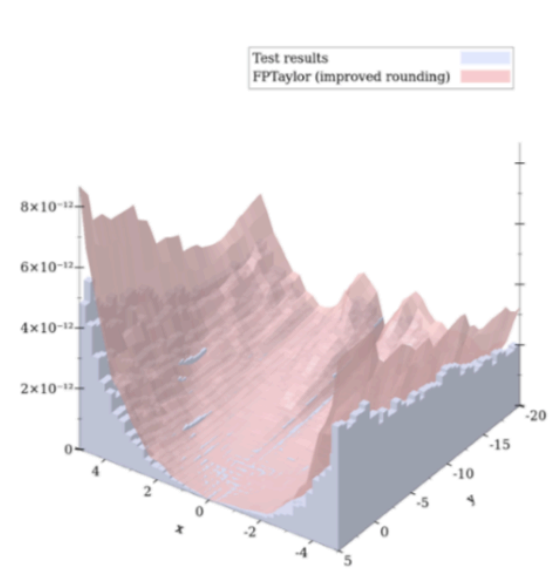
Rounding error while adding x and y in [-1, 2]

Background:
One seeks Tight and Rigorous upper bounds

[a]From *Sound Compilation of Reals* by E. Darulova and V. Kuncak
[b]Only the approximate function is shown

```
double jetEngine(double x1, double x2) {
  double t = 3 * x1 * x1 + 2 * x2 - x1
  double q = x1 * x1 + 1
  double p = t / q
  double s1 = 2 * x1 * p * (p - 3)
  double s2 = x1 * x1 * (4 * p - 6)
  double s3 = 3 * x1 * x1 * p
  double s4 = x1 * x1 * x1
  double s5 = 3 * p
  return x1 + ((s1 + s2) * q + s3 + s4 + x1 + s5)
}
```

Deriving such error functions for very large Floating-point expressions can be quite handy! How to do this?



Alexey Solovyev, Marek S. Baranowski, Ian Briggs, Charles Jacobsen, Zvonimir Rakamaric, Ganesh Gopalakrishnan:
Rigorous Estimation of Floating-Point Round-Off Errors with Symbolic Taylor Expansions. ACM Trans. Program. Lang. Syst. 41(1): 2:1-2:39 (2019)

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Question 1: Onto Scalable Roundoff Estimation

- Rigorous roundoff analysis methods have had limited scalability
- SATIRE offers enhanced scalability
  - to a point that more meaningful insights can be obtained
    - Could have helped our mixed-precision work on FPTuner (POPL'17) scale
- Has many strengths over today's empirical alternatives
  - Shadow-value based
  - Statistical
- Exceeds the best tools in its class in terms of scalability

Question 2: Scalable Reproducibility

- The numerical behavior of a program can change upon porting
- Sources:
  - Different IEEE rounding, presence of FMA, …
  - Non-IEEE optimizations
  - Falling into C-undefined behaviors
  - Truly ill-conditioned code
  - Compiler bugs
- Need: A tool that can help root-cause non-reproducibility
- Our tool FLiT is promising in this regard
  - It is a search-based tool that is empirical
    - finds repro issues for the test inputs chosen

Question 3: What are the Resilience Concerns?

- Bit-flips can destroy results
  - So can polyhedral compiler bugs
- But what is a “golden answer” ?
- Is there a way to detect bit-flips / compiler bugs without knowing these golden answers ?
- Traditional approaches
  - DMR : code duplication
- Our approach
  - Predict “virtually roundoff-free values” to appear “T steps ahead”
  - FPDetect is promising in this regard

Organization of the rest of the talk

- High-level details of
  - Satire,
  - FLiT, and
  - FPDetect
- How to bring the communities closer…
  - Papers on FP analysis appear in “PL conferences” and “HPC conferences”
  - They have a different take on things, different criteria for rigor, etc.

Scalable Abstraction-guided Tool for Incremental Reasoning of floating-pt Error

Tools similar to SATIRE

- Gappa ("error = diff between less accurate and more accurate" – G. Melquiond – courtesy F. de Dinechin)
- Rosa
- Real2Float
- Precisa
- FPTaylor (best in the class)
- … (see FPTaylor TOPLAS'19 paper for others)

- SATIRE focuses on first-order error (sensitivity study in our arxiv)
  - Allows it to scale to > 1M operator nodes
  - Comparable rigorous tools handled only about 100 nodes
  - Satire's bounds are almost always tighter (ignoring second-order error)

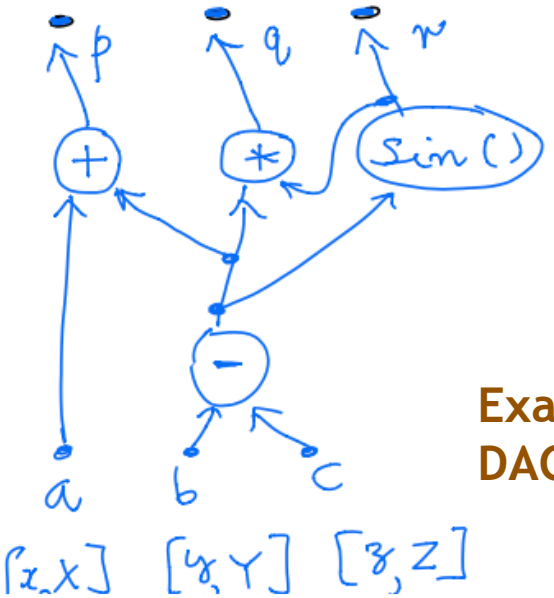
SATIRE in one slide

A
Large
Expression
DAG

Intervals
of values
For each
Input of the
Expression DAG

Maximum
Absolute
Error
across all
points in the
input intervals

"not just
interval analysis"
(more like
"symbolic
affine")



Example
DAGs

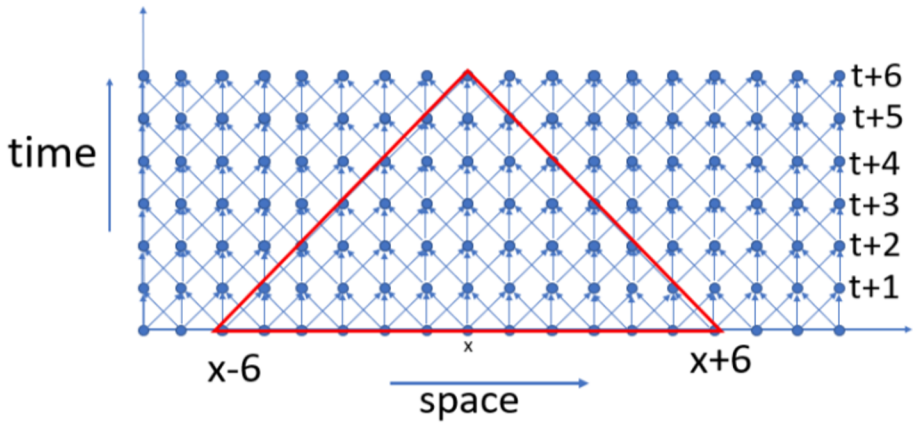


Fig. 1. Simplified 1D stencil over 6 time steps

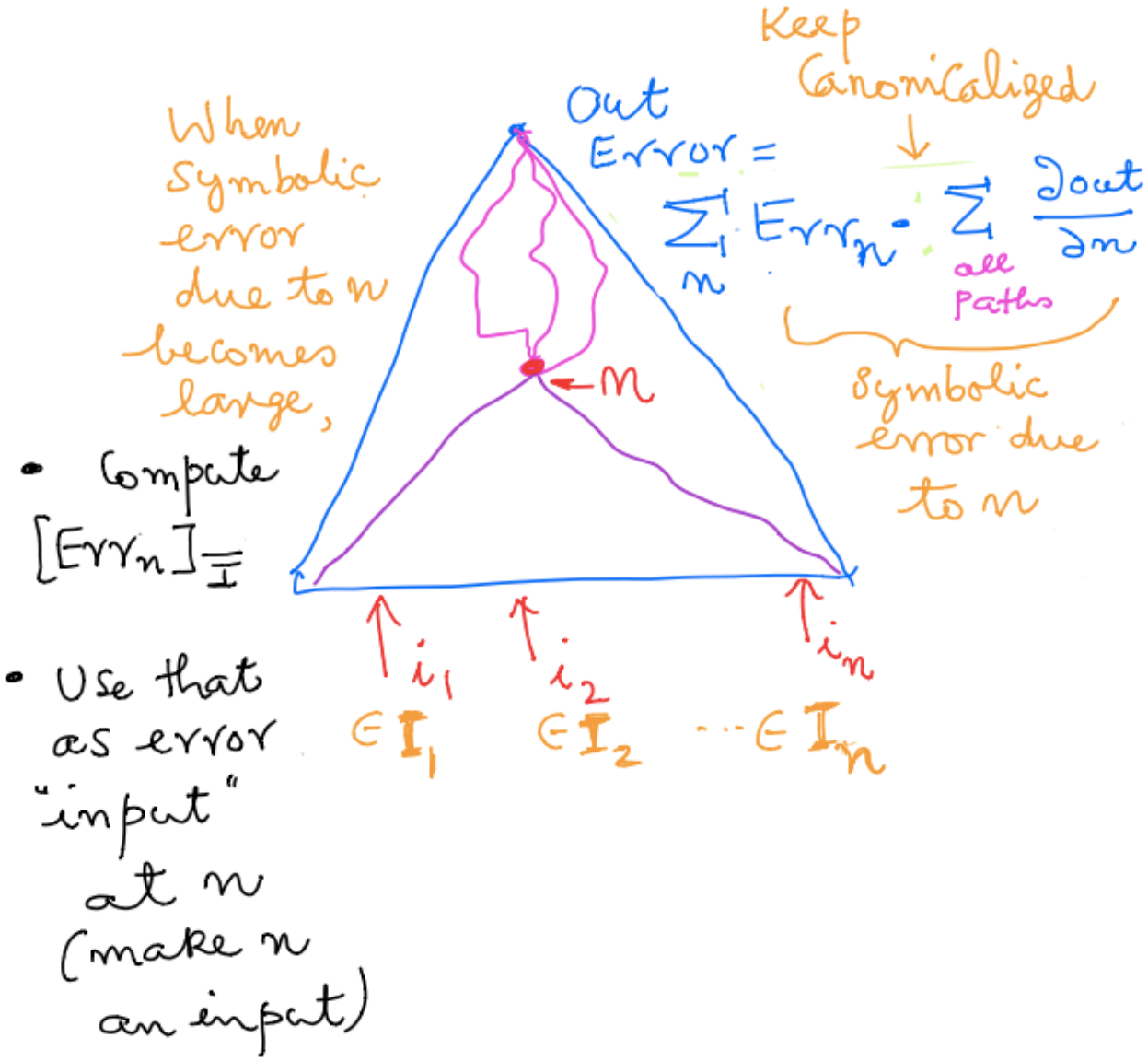
Examples studied using SATIRE

- Unrolled loops coming from
  - PDE solvers
    - Stencils
  - Others
    - Mat-Mat
  - Of special interest
    - Scan
    - FFT -- SATIRE's bounds are better than published analytical bounds
- Tricky ones
  - Lorenz equations

- Why unroll loops?
  - Finding tight loop invariants is very hard (for FP code)
  - Unrolled loop analysis can lend insights
  - Just to obtain meaningful large expression DAGS (and face the scaling challenges!)

How SATIRE works

- Symbolic reverse-mode A/D
  - Derivative-strength of Out wrt n
- Keep expressions canonicalized
- Multiply forward error at n
- Compute incrementally
- Abstract when Err_n becomes large



Scalability-Related Lessons Learned

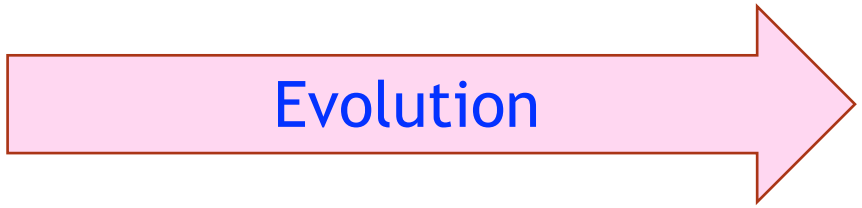
- Scalability is a function of
  - Amount of Non-linearity
  - Number of reconvergent forward paths
  - Examples
    - FDTD goes to ~**200K** operators (without abstractions)
    - Lorenz system: Bottlenecked at ~**200** operators (without abstractions)
- Good canonicalization is key (we win over other tools due to this)
- Good abstraction heuristics are key (Shannon-Info measure used by us)
- Symbiotic uses of SATIRE with Empirical tools is a promising path
  - Have developed a promising method to estimate relative error
- Ability to extract and publish specifications can be a good practice!

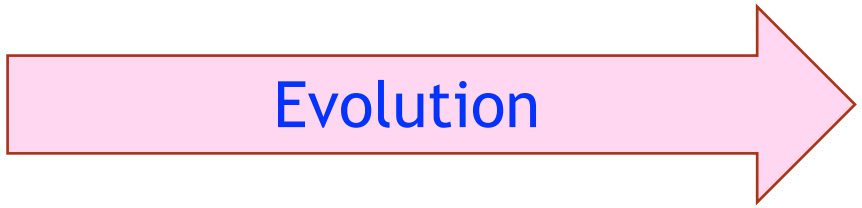
A Floating-point Litmus Tester

FLiT helps keep Science on Keel when Software Moves

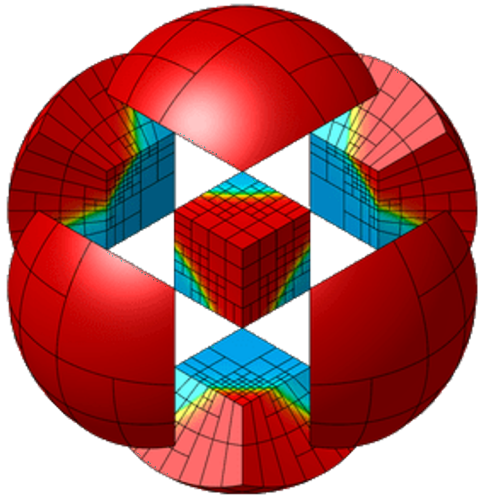
- **Yesterday's results** 

Results a decade later

- **Compilers** 

- **Hardware** 

A Medium-Sized Example: MFEM



source files	97
average functions per file	31
total functions	2,998
source lines of code	103,205

- Open-source finite element library
  - Developed at LLNL
  - https://github.com/mfem/mfem.git
- Provides many example use cases
- Represents real-world code

MFEM Results displayed after FLiT-search

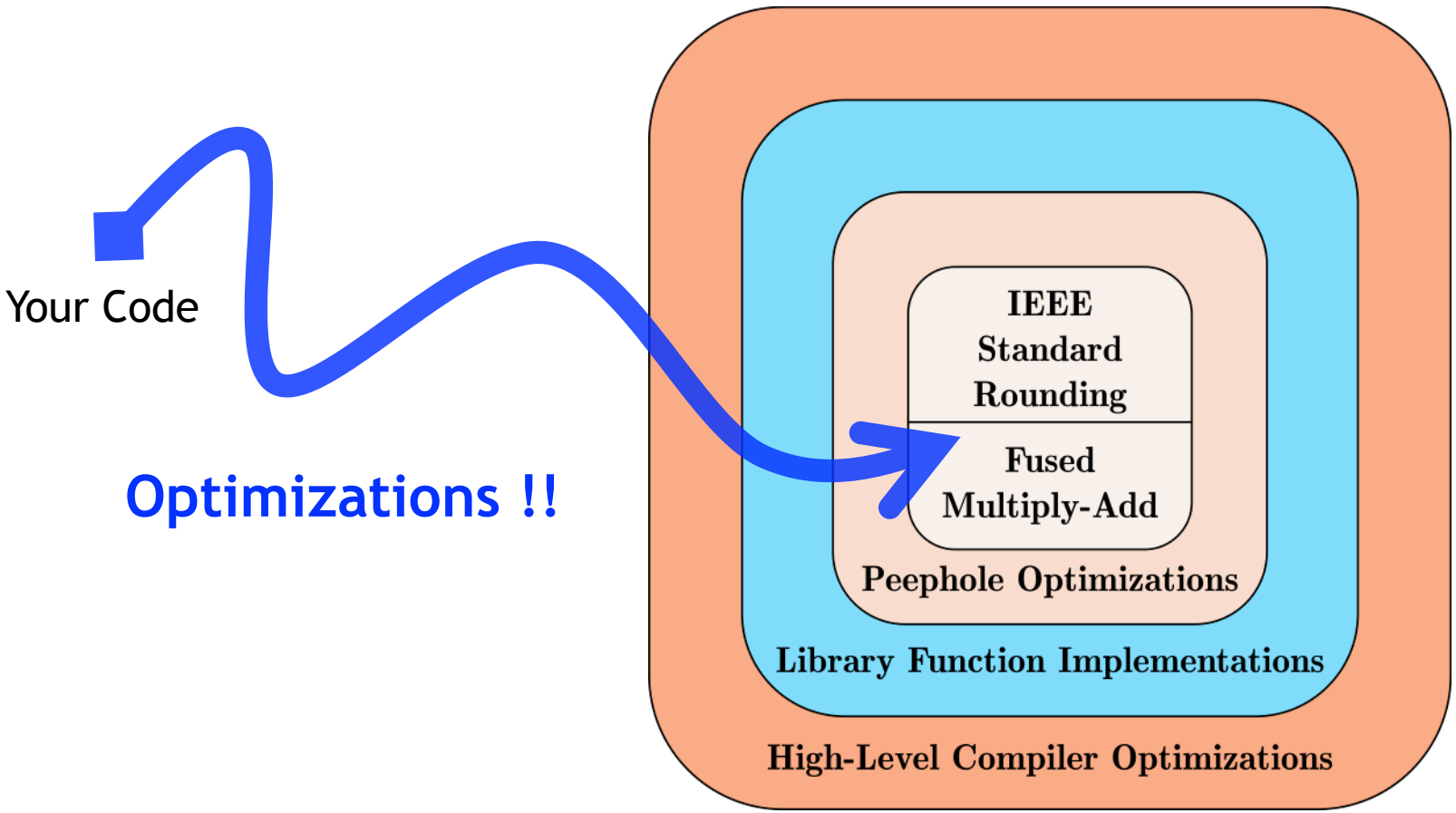
```
sqlite> select compiler, optl, switches, comparison, nanosec from tests;
compiler     optl        switches     comparison  nanosec
-----------  ----------  -----------  ----------  ----------
clang++-6.0  -O3         -ffast-math  0.0         2857386994
clang++-6.0  -O3         -funsafe-ma  0.0         2853588952
clang++-6.0  -O3         -mfma        0.0         2858789982
g++-7        -O3         -ffast-math  0.0         2841191528
g++-7        -O3         -funsafe-ma  0.0         2868636192
g++-7        -O3         -mfma        193.007351  2797305220
sqlite> .q
```

One compilation had 193% relative error (also ran faster!)

Other compilations had no error.

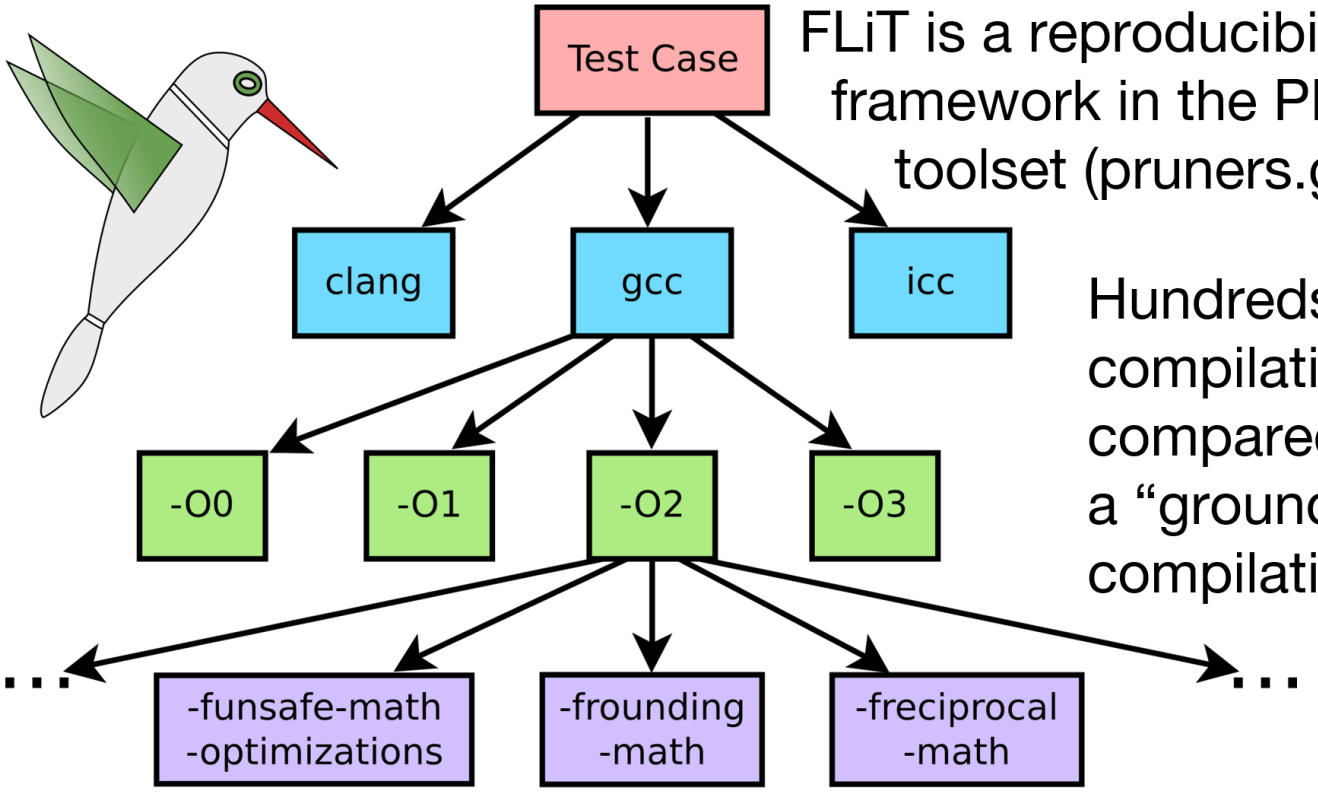
Which site in the source-code is responsible?

The code you run is not what you ”see”



We offer FLiT, a tool to study FP optimizations

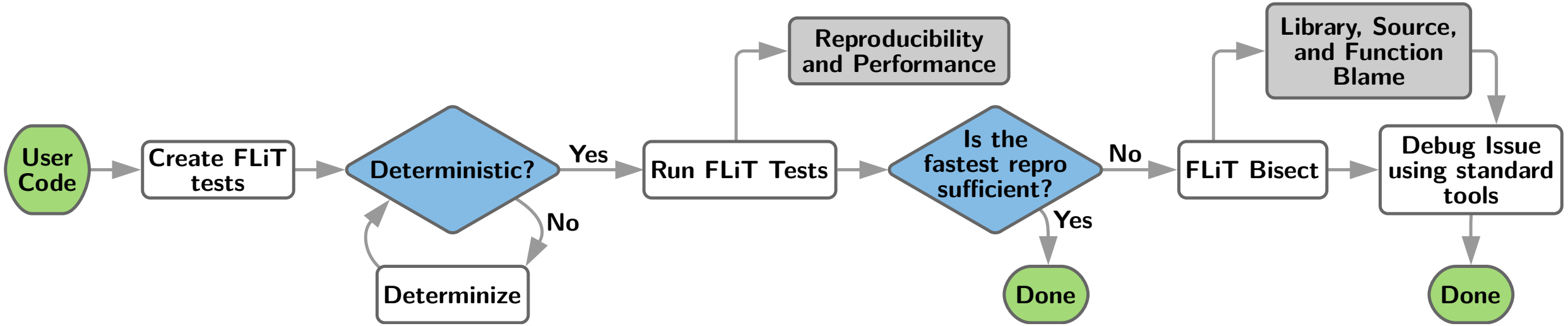
FLiT: Part of the PRUNERS Toolset



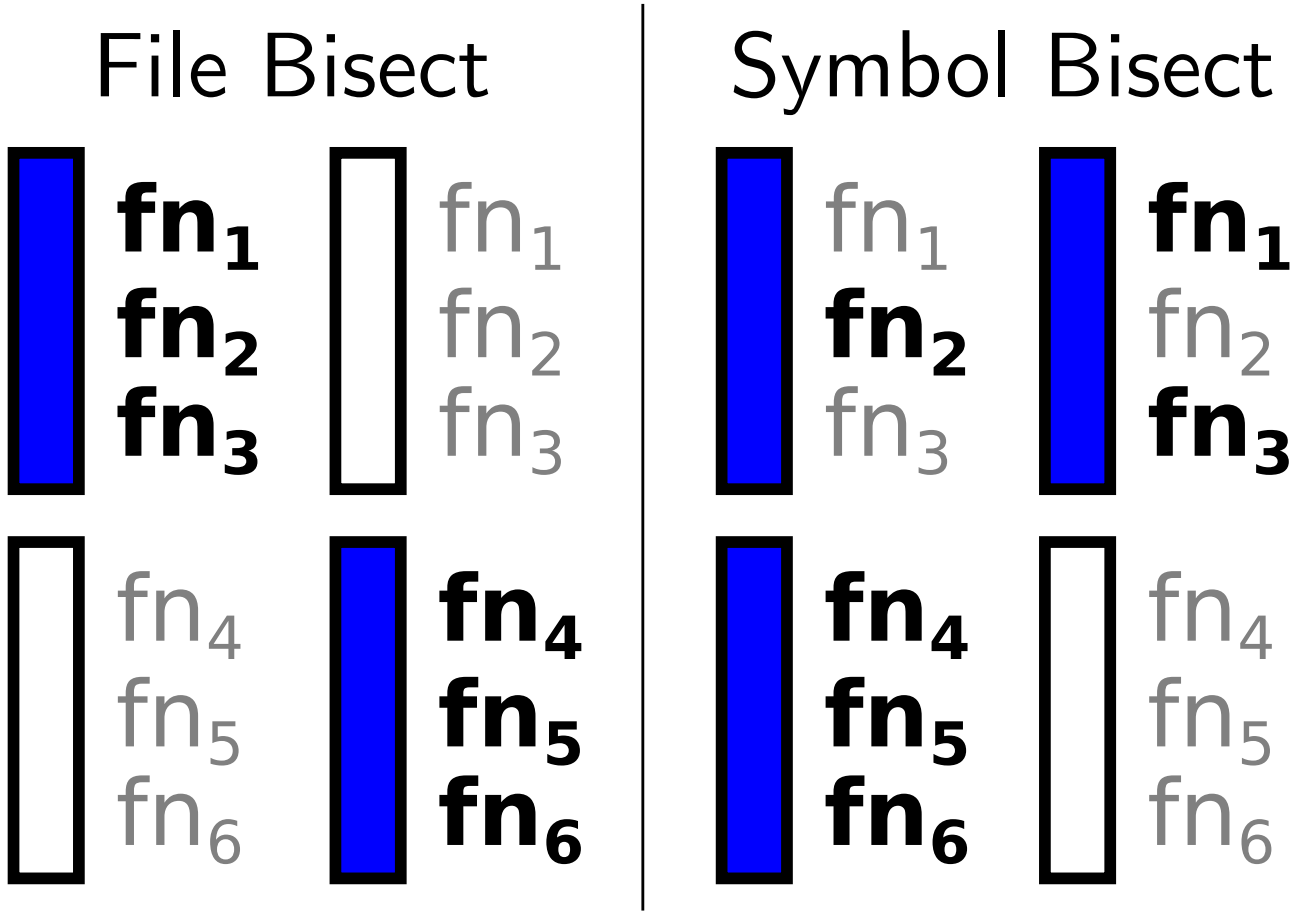
FLiT is a reproducibility test framework in the PRUNERS toolset (pruners.gihub.io).

Hundreds of compilations are compared against a “ground-truth” compilation.

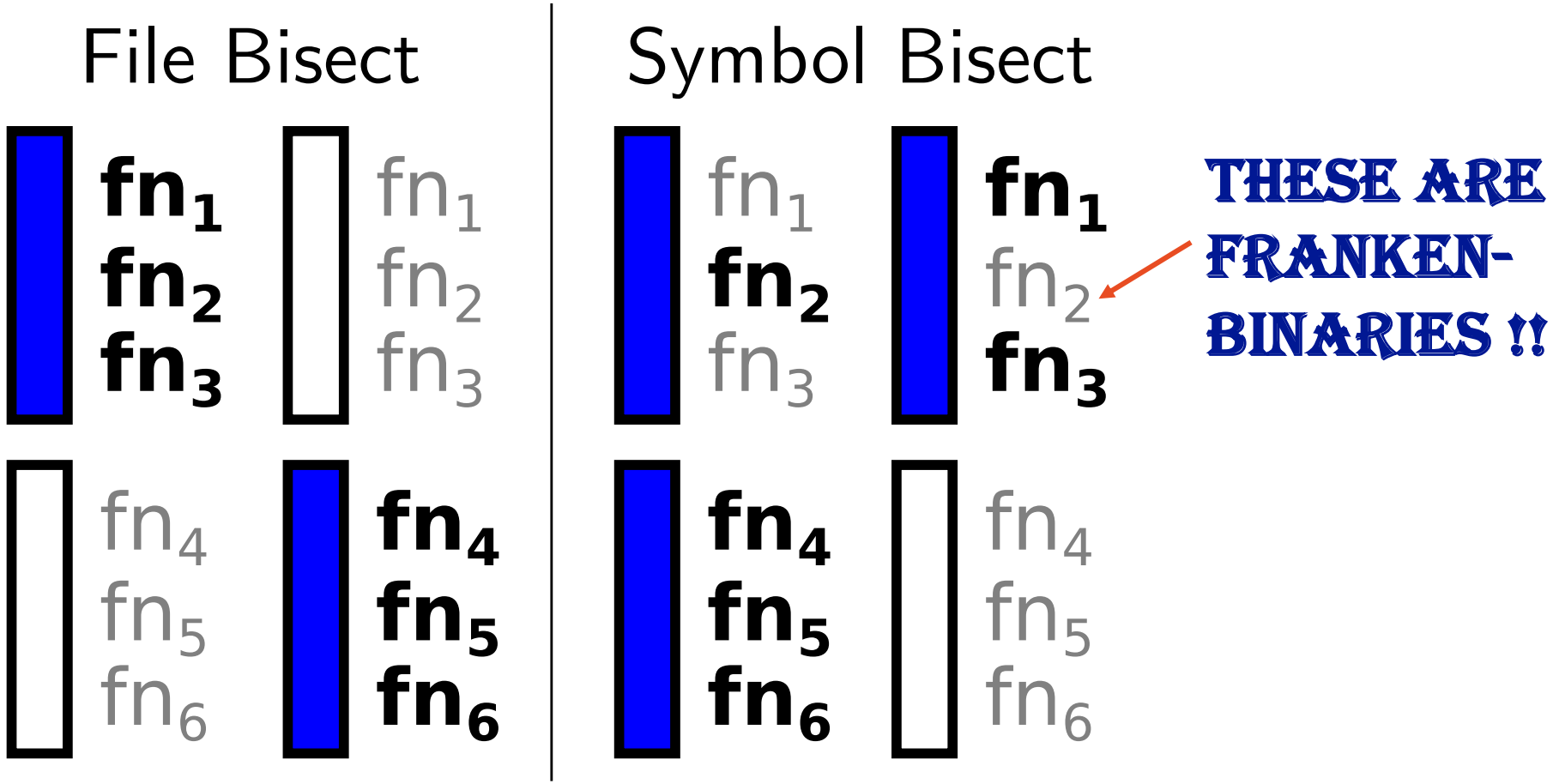
FLiT Workflow



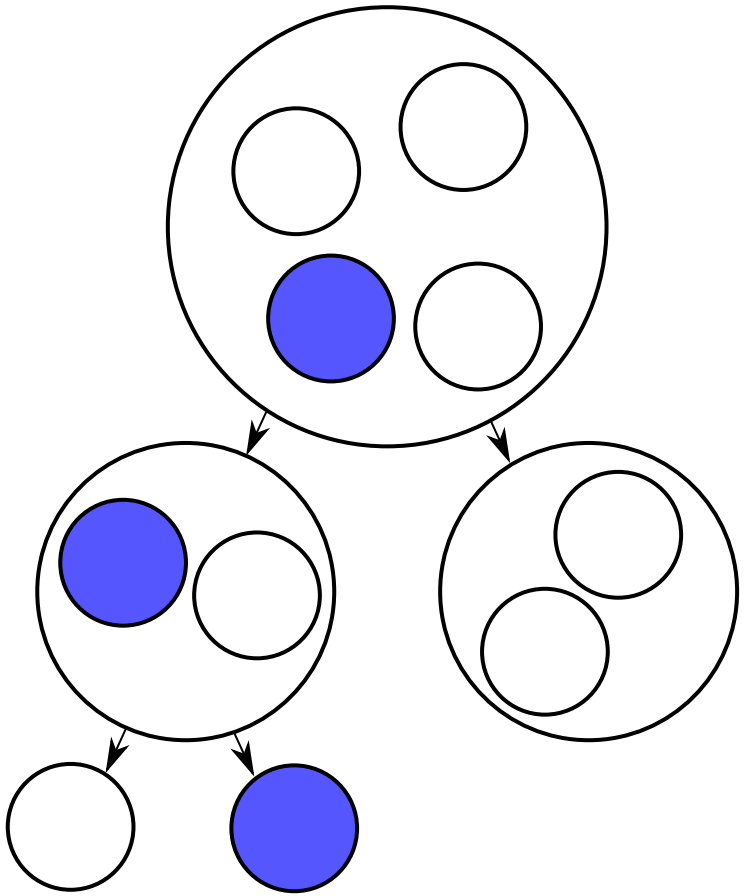
FLiT Bisect: File or Symbol

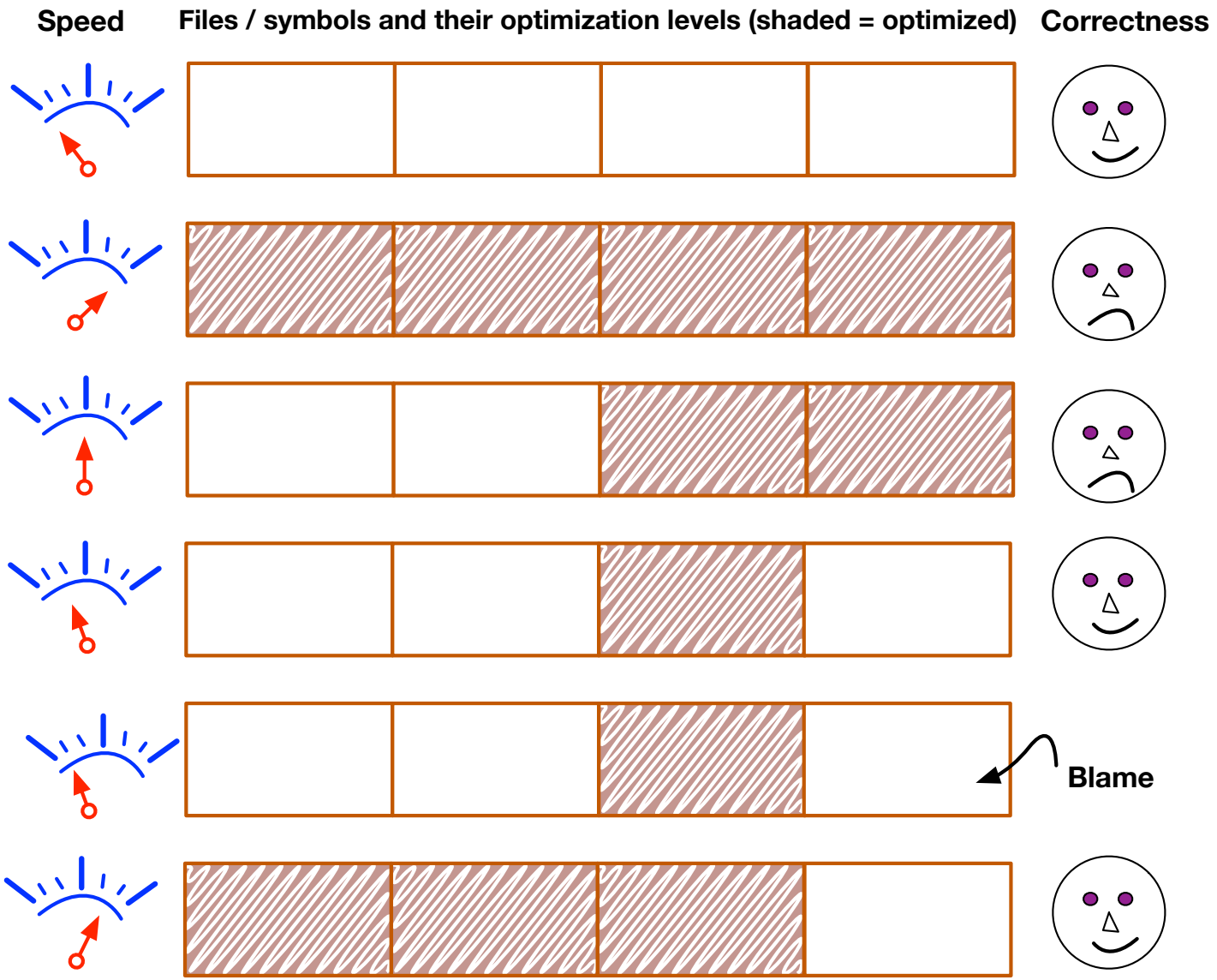


FLiT Bisect: File or Symbol



FLiT-Bisect under Singleton Minimal Set





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Significant “finds” using FLiT

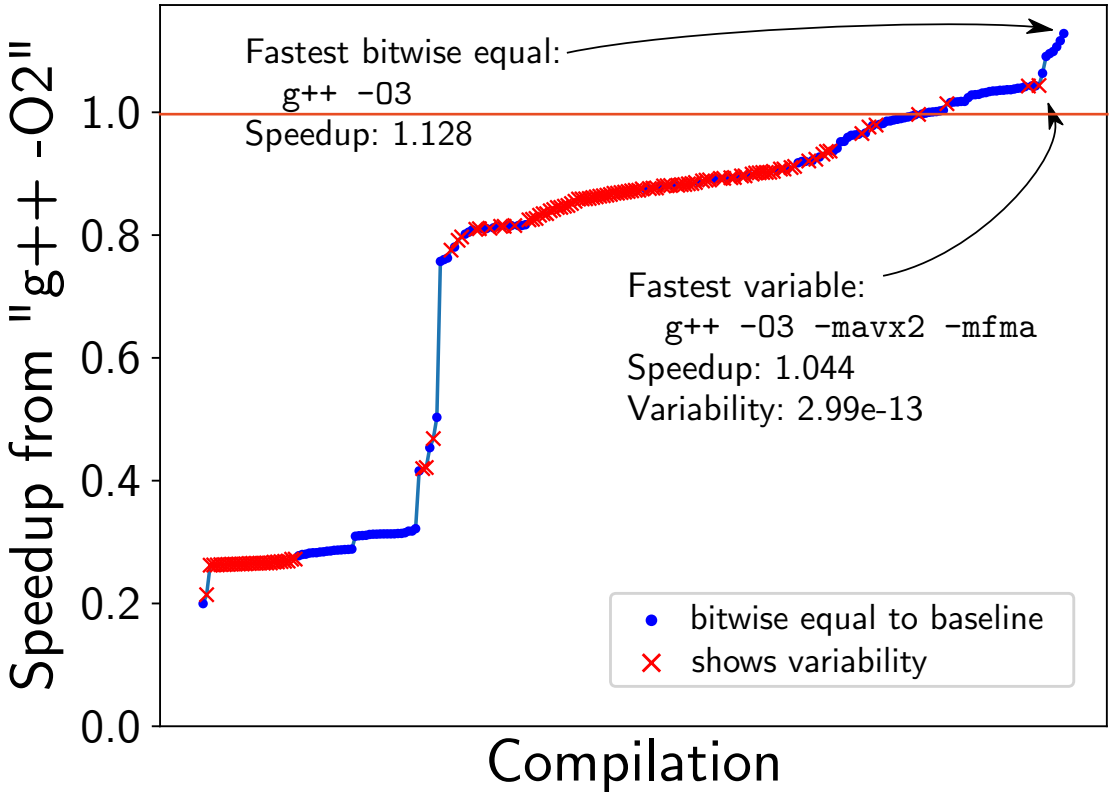
- Test-13 within the MFEM library
- Compiler optimization that involved
  - AVX2, FMA, Higher precision intermediate floating-point values
- Result has 190% relative error
- Symbol-based bisection narrows down problem
  - Single function calculating $\mathbf{M = M + a \cdot A \cdot A^T}$
- Conversation with developers underway
- Significant Recent Milestone:
  - Recently a 1M LOC critical export-control code was analyzed by FLiT and we found issues (root-caused to a small number of LOC)
    - FLiT recently deployed on LLNL machines (contact Mike Colette, Olga Pearce, …)

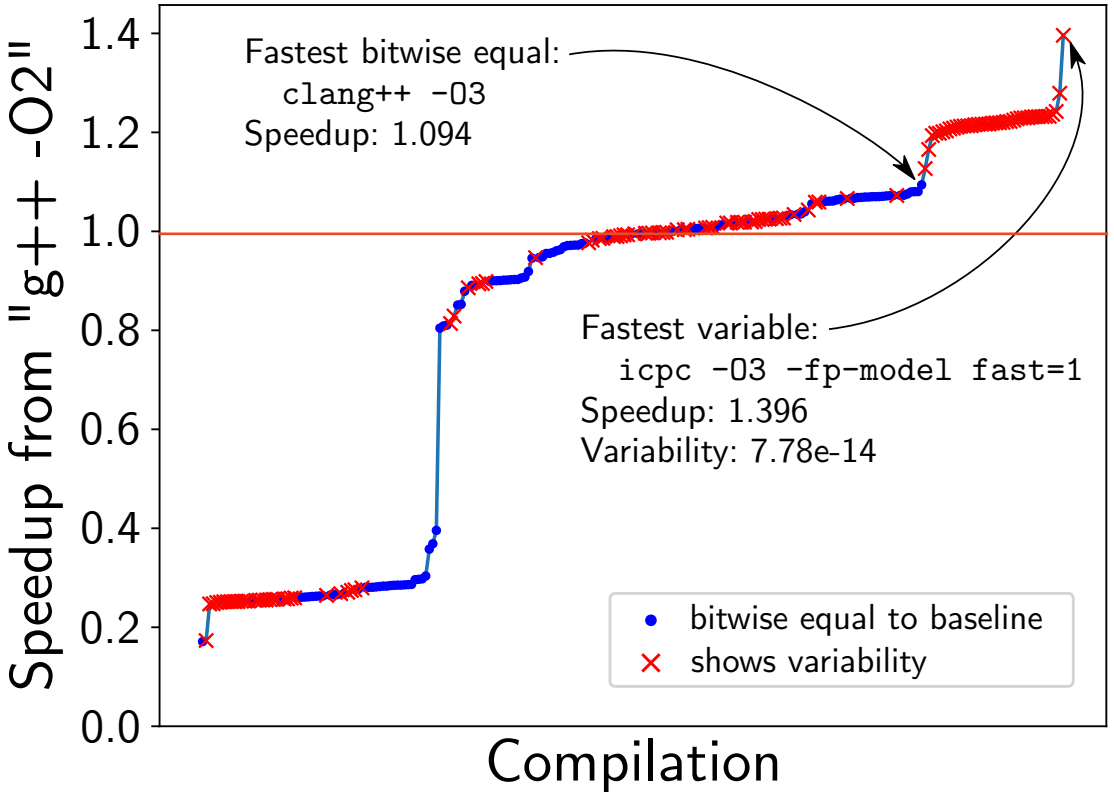
Interesting Find: Non-Portability Between IBM XLC and GCC

```
#define xsw(a,b) a^=b^=a^=b
```

Nan in IBM XLC ; “OK value” in GCC ☺

Compilation, Repro, Speedup (Two MFEM Tests)





FPDetect:
Using the computed tight rounding-error bound to detect “unexpected numerics”

System Resilience: Need

Courtesy: IBM Tech Journal 2006, DOI:10.1109/MDT.2009.153

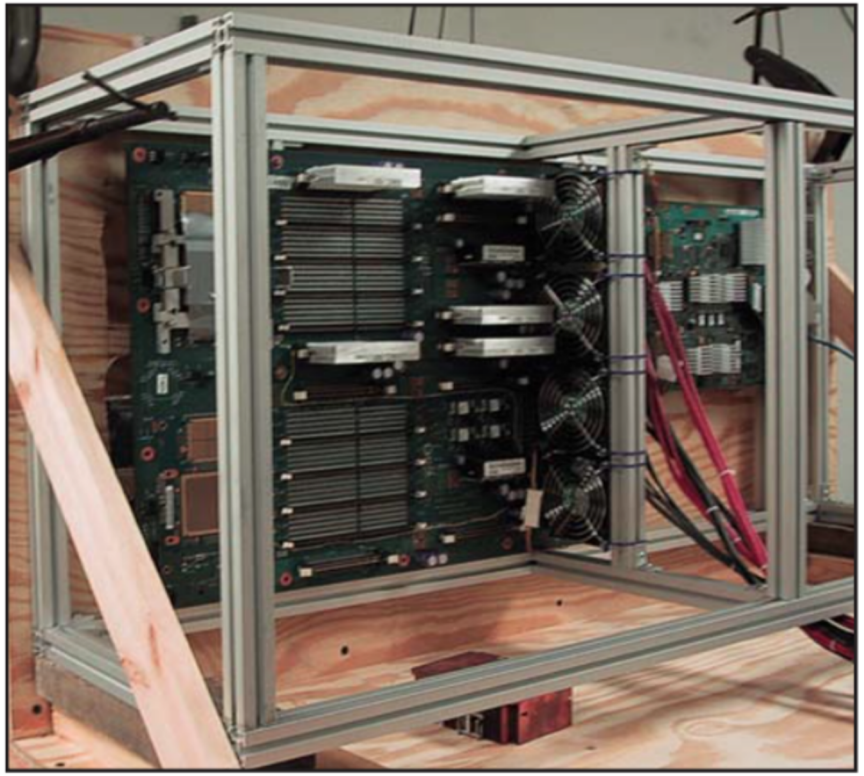


Figure 2

POWER6 test system mounted in beamline.

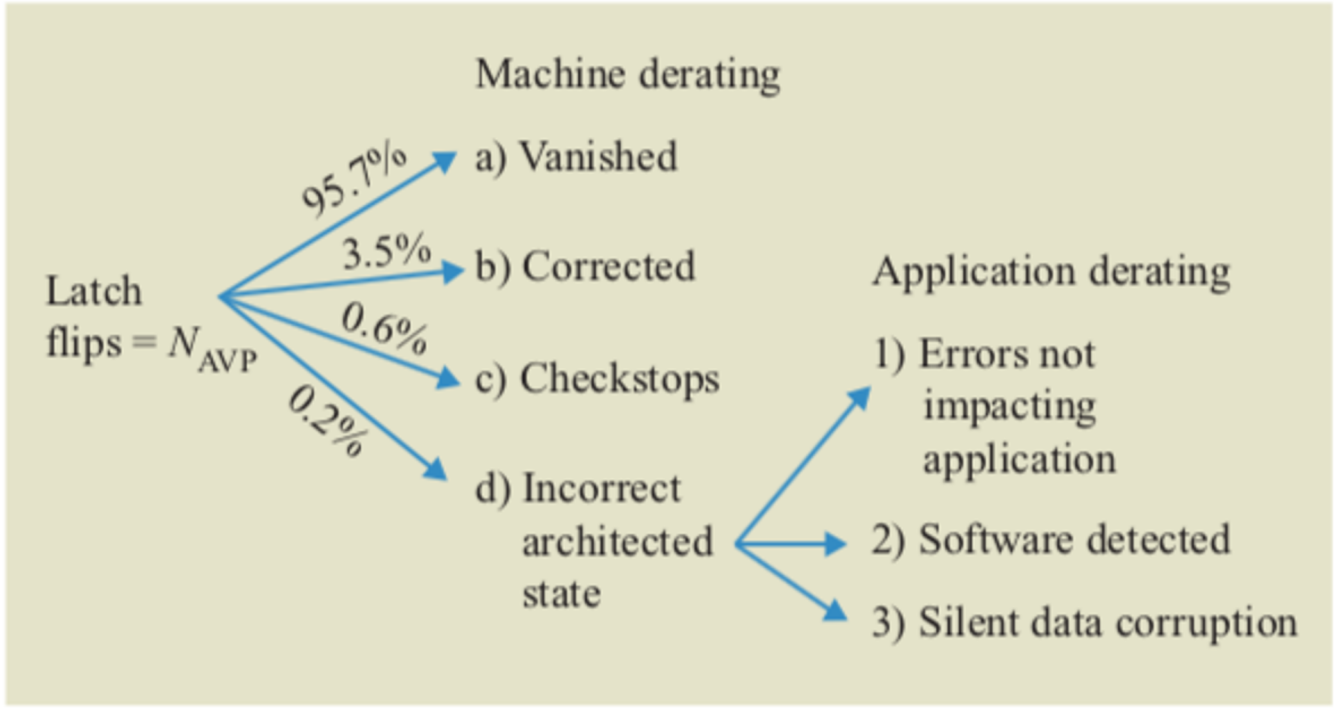


Figure 1

Taxonomy of derating terms.

Why is System Resilience (research/deployment) Stymied?

- Nobody wants slowdown ("bit-flips == fake-news")
  - Myopic when facing end-of-Moore
    - P. Bose of IBM, Engelmann, Cappello, …. many other other studies
- No good error detector!
  - High overhead
  - Omissions
  - False positives
- Way forward
  - Focus on specific aspects
  - Cover that well with
    - **Rigorous Guarantees**, High Detection, Low Overhead, **No False Positives**

Our contributions

- What is FPDetect?
  - Method to detect errors in the "data space"
    - **Currently for Stencil code**
  - Distinguishes "normal data" from SDC-laden data

- How does FPDetect work?
  - Infers round-off error as a specification
  - If observed value aberration is more than inferred, then "something else is going on", which could be
    - Bit flips
    - Compiler bugs

FPDetect Mental Model

- “Skate where the puck will be”
  - W. Gretsky

Predict “almost Real” value at t+6

- When “computation reaches” t+6, check modulo rounding loss

(“Error = diff between less accurate and more accurate” – G. Melquiond , courtesy F. de Dinechin)

  - We have omitted details that show that we can fire the detectors infrequently and still guarantee coverage across a whole protected volume

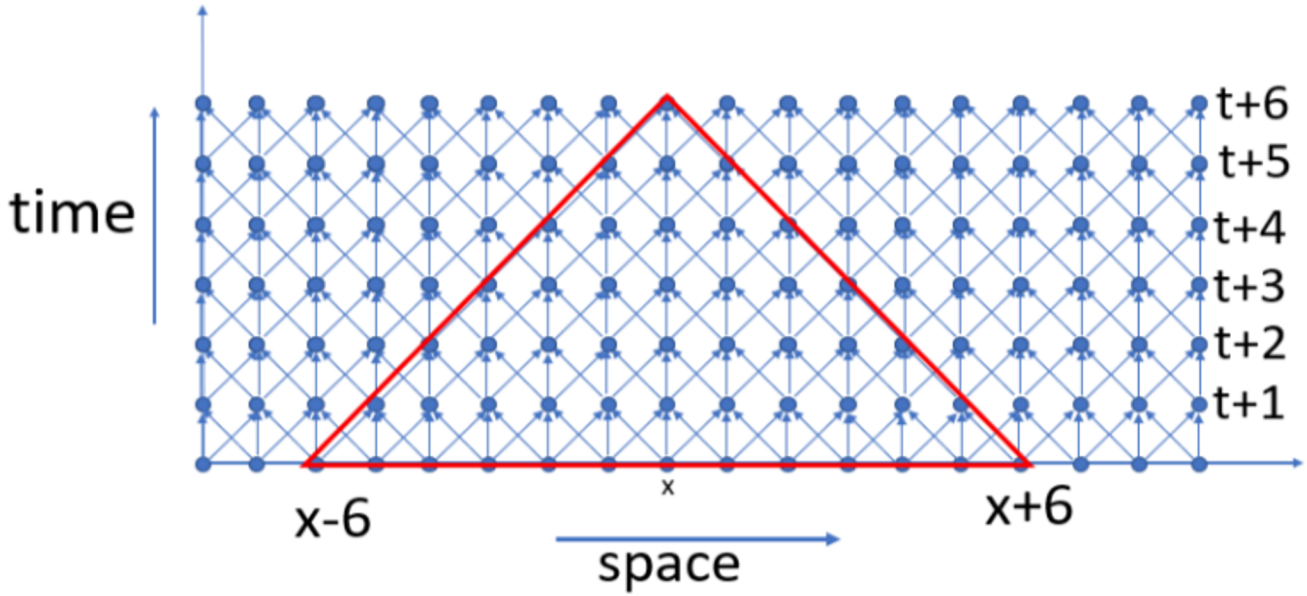


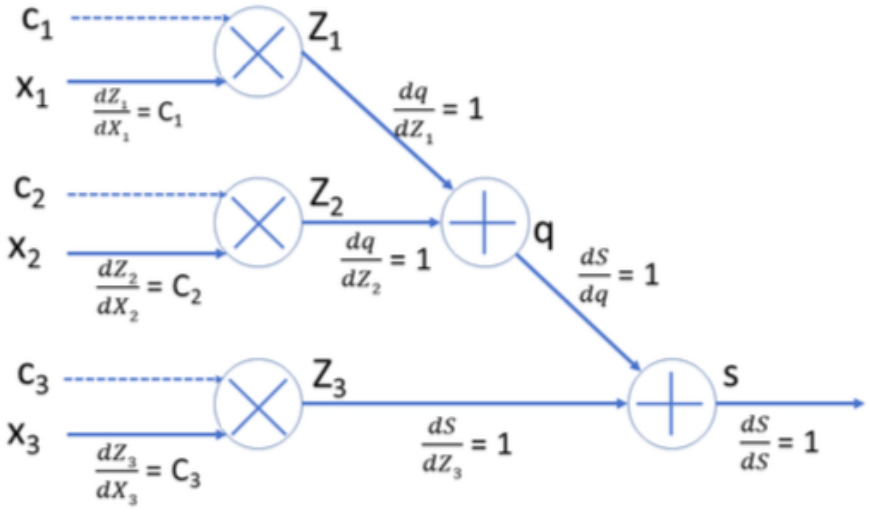
Fig. 1. Simplified 1D stencil over 6 time steps

Obtaining “almost Real” value T-steps ahead

- Compute the “Expected Answer” based on full path-strength
  - evaluated through “unfolding”,
  - implemented using Kahan’s sum
  - vectorization to minimize overheads

Table 1. Evolution of Coefficient values as we unroll the stencil

Set Number	Coefficients at Offsets						
offset index →	-3	-2	-1	0	1	2	3
Set_1	0	0	0.25	0.5	0.25	0	0
Set_2	0	0.0625	0.25	0.375	0.25	0.0625	0
Set_3	0.015625	0.09375	0.234375	0.3125	0.234375	0.09375	0.01562



Error Detection in the Abstract

- Challenges overcome in our analysis
  - tightly estimate error across all reconvergent paths
  - eliminate paths that have been dominated by others
    - Passes our empirical checks … but …
    - Need: Formal Verification tool that can check our “ferocious” hand calculations
      - Why3 ?

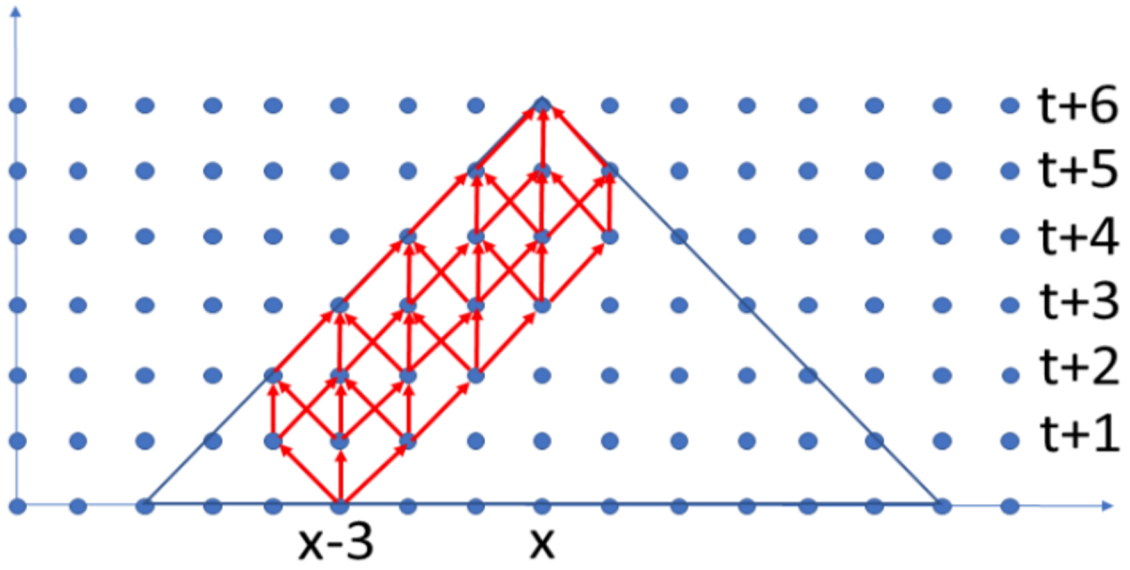


Fig. 2. Illustration of path dominance

FPDetect for Logical Bug Detection

Table 3. Software bug detection results.

	Loop bound			Array access				Loop bound			Array access		
	#SL	#RL	%det	#Sl	#Rl	%det		#SL	#RL	%det	#Sl	#Rl	%det
H1	374	374	100	300	267	99	H2	370	370	100	300	258	100
H3	374	374	100	300	267	98	H4	486	106	99	300	26	100
H5	486	106	100	300	26	100	H6	486	106	100	300	26	100
P1	10	10	100	18	17	100	P2	12	12	100	18	17	100
P3	12	12	100	18	17	100	P4	370	310	100	300	223	87
P5	370	310	96	300	223	82	P6	370	310	96	300	223	82
P7	414	104	98	300	48	97	P8	416	104	100	300	44	98
P9	416	104	98	300	44	98	W1	360	280	100	300	215	55
W2	360	280	44	11	6	54	W3	360	350	31	300	266	45
W4	260	280	45	300	215	60	W5	360	280	53	300	215	62
W6	360	350	100	300	266	100	C1	360	360	100	300	262	100
C2	360	360	100	300	262	100	C3	360	360	100	300	262	100

We found that comparable soft-error detectors (SSD and AID) do not serve as effective logical-bug detectors

Summary

- Rigorous method Satire (see arxiv paper) for error analysis
  - May be used symbiotically with Empirical tools for relative-error estimation, informing where to tune, etc.
- Empirical tool FLiT (see HPDC'19, CACM forthcoming)
  - Needs Rigorous methods to define coverage to be above "just luck"
- Rigorous tool FPDetect (TACO, accepted)
  - Uses rigorous methods, and has spawned empirical precision profiling methods
    - Learn by how much the FPDetect detector is pessimistic wrt. the shadow-value
    - Learn how much precision exists in a bounding volume
    - At run-time, do not use any shadow-value (only the detector)
    - Give report of "true" precision in bounding volume

Plea

- More rigorous tools needed
  - More bridges between Rigorous and Empirical needed
    - Helps scale
    - Helps tech-transfer

More workshops like this needed!